Equilibrium of a Rigid Body Under Coplanar Forces

Week 3, Lesson 1

- Coplanar Forces
- Second Condition of Equilibrium
- The Moment or Torque
- Two Conditions of Equilibrium

References/Reading Preparation:

Schaum's Outline Ch. 3

Principles of Physics by Beuche – Ch.4

Equilibrium – Coplanar Forces

An object may be in equilibrium under ***concurrent forces***, but **not in equilibrium** under ***coplanar forces.***

Recall:

> ***Concurrent Forces*** are forces whose line of action all pass through a common point.

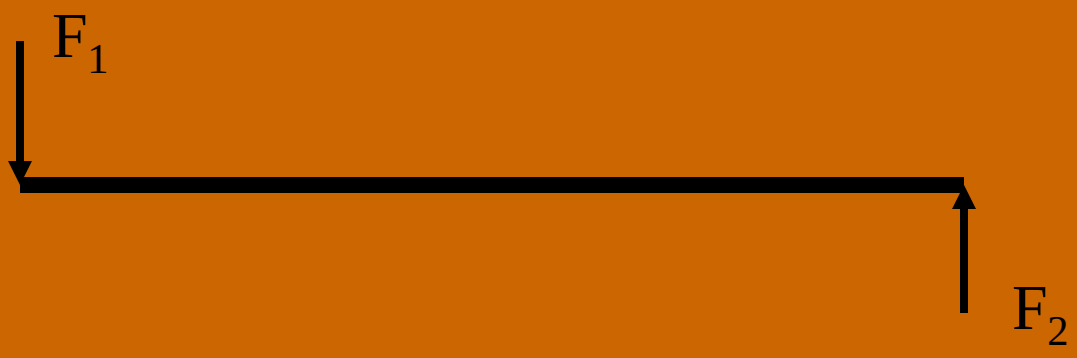
Coplanar forces are forces that are on the same plane but whose line of action **does not** pass through the same point.

Under this situation, there is a second condition that must be satisfied if an object is to be in Static Equilibrium.

Second Condition of Equilibrium

There is another condition involving ***rotation*** that must also be satisfied.

Consider the following situation:



If $F_1 = F_2$, the sum of the forces in both the x- and y- directions = 0

BUT… Is it really in equilibrium??… NO!

IT WILL SPIN AROUND AND AROUND…

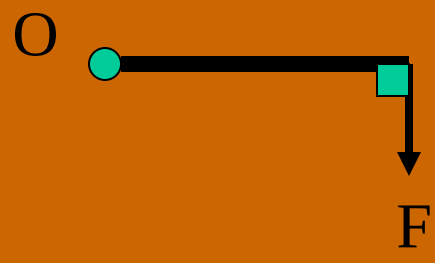
The Moment (or Torque)

Let's look at this a bit closer:

Consider an object…

subjected to a force F….

acting perpendicular to the object.

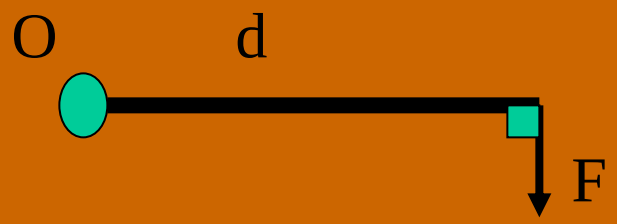


This force will cause a twisting (or turning) action to happen about a point O.

We call this action a ***Moment (M)***, or ***Torque (τ)***.

The Moment (or Torque)

Coming back to our diagram:



If the distance from the point O to the force is distance ‘d’,

Then, the moment M = the force F times the distance d.

$M = F \times d$ Where: M = moment (in N•m) or torque τ
F = the force (in Newtons – N)
d = the perpendicular distance from the point of rotation to the force in metres)

The Torque

Since the Moment can also be referred to as the Torque,

Then,

$\tau = F \times d$

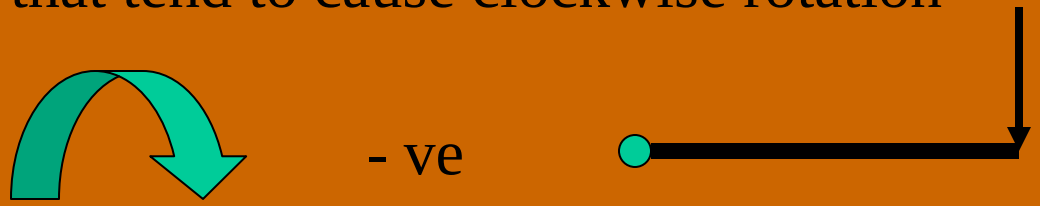
where:

τ = Torque in N•m
F = force (N)
d = the perpendicular distance (m)

The distance ‘d’ can also be referred to as the Lever Arm.

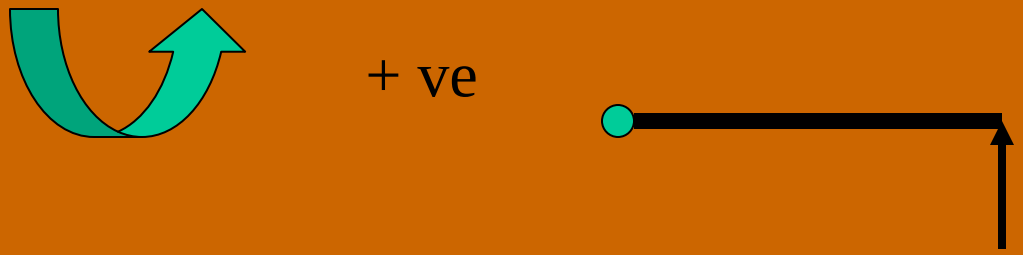
About Torques and Moments

We call the torques that tend to cause clockwise rotation negative.



And,

We call the torques that tend to cause counterclockwise rotation positive.



The Two Conditions for Equilibrium

The two conditions for equilibrium of a rigid object under the action of coplanar forces are:

1) The ***first***, or ***force condition:***

The vector sum of all forces acting on a body must be zero

$$\Sigma \mathbf{F}_x = 0 \text{ ; and } \Sigma \mathbf{F}_y = 0$$

where the plane of the coplanar forces is the xy-plane

2) The ***second***, or ***torque condition:***

The sum of all the torques (or moments) acting on the object must be zero:

$$\Sigma \tau = 0$$

Let’s do some examples to show:

1) How to calculate a torque (or moment)

2) That the entire weight of an object can be considered to act through its centre of gravity.

3) That a single vertically upward directed force, equal in magnitude to the weight of the object and applied through its centre of gravity, will keep the object in equilibrium.

4) That if the sum of the torques is zero about one axis for a body, it is zero for all other axes parallel to the first.